** 

 IV Wojewódzki Konkurs

 **„Matematyka z kalkulatorem graficznym”**

*ZSDiOŚ im. Jana Zamoyskiego w Zwierzyńcu*

Eliminacje

marzec / kwiecień 2019r.

GODZINA ROZPOCZĘCIA: 11.00

CZAS PRACY: 90 minut

LICZBA PUNKTÓW: 50

**Instrukcja dla piszącego**

1. Rozwiązania zadań i odpowiedzi wpisuj w miejscu na to przeznaczonym.
2. Pisz czytelnie, używając czarnego (niebieskiego) długopisu lub pióra.
3. Możesz korzystać tylko z cyrkla, linijki oraz kalkulatora graficznego przygotowanego

 przez Komisję.

1. Pamiętaj, jeśli nie potrafisz rozwiązać zadania za pomocą kalkulatora, rozwiąż je w sposób tradycyjny.
2. Staraj się nie wpisywać tylko samych wyników, ale również sposób rozumowania

 (w tym obliczenia) prowadzący do rozwiązania zadania.

1. W przypadku, gdy w wyznaczonych na rozwiązanie kratkach zabraknie miejsca, możesz wykorzystać do zapisania dalszej części rozwiązania, odwrotną stronę bieżącej kartki.

**Imię i nazwisko: ………………………………………………………………………………..**

**Szkoła Podstawowa…………………………………………………………….........................**

Życzymy powodzenia

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**ZADANIE 1 (4pkt)**

**a)** Dane są liczby : *p* = $\frac{2\sqrt[3]{1296}-4\sqrt[3]{625}}{\sqrt{2}(3\sqrt[3]{6}-5\sqrt[3]{5})} i\frac{}{}ii ,ik $*q* = $(\sqrt[3]{2}-\sqrt[3]{3})∙(\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9})\frac{}{},$

Zapisz dokładne wartości liczb *p* i *q*.

**b)** Wpisz w przygotowanej tabelce pięć kolejnych cyfr **po przecinku** przybliżenia dziesiętnego liczby *a,* która jest liczbą przeciwną do wartości bezwzględnej z różnicy liczb *p* i *q*.

**Rozwiązanie: a)**

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**Odpowiedź: a)**  *p =* *q=*

**Rozwiązanie: b)**

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 **Odpowiedź: b)**

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**ZADANIE 2 (5pkt)**

Oblicz dokładną wartość wyrażenia: $\frac{NWD\left(140,567\right)-NWW(140,567)}{NWD\left(600,72\right)+NWW(600,72)}$ .

Uwaga: *NWD(x,y)* oznacza największy wspólny dzielnik liczb *x* i *y*, zaś *NWW(x,y)* oznacza najmniejszą wspólną wielokrotność liczb *x* i *y*.

**Rozwiązanie:**

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**Odpowiedź:** …………………………………………………………………………………….

**ZADANIE 3 (6pkt)**

Dane są cztery, kolejne, dodatnie liczby: *a*, *b*, *c* i *d*. Druga liczba jest dwa razy większa od pierwszej, trzecia liczba jest trzy razy większa od drugiej, a czwarta liczba jest cztery razy większa od trzeciej.

**a)** Oblicz sumę tych liczb wiedząc, że pierwsza z nich jest równa 4*x* + 8y - 12z - 16p.

**b)** Oblicz średnią arytmetyczną tych liczb. Wynik podaj w najprostszej postaci.

**Rozwiązanie:**

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**Odpowiedź** …………………………………………………………………………………………………...

**ZADANIE 4 (6pkt)**

Wyznacz wszystkie liczby naturalne  *n* mniejsze od 60, dla których wartość wyrażenia $\frac{3n-1}{11}$
jest liczbą całkowitą.

**Rozwiązanie:**

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**Odpowiedź:** …………………………………………………………………………………….

**ZADANIE 5 (4pkt)**

W oknie **Geometry**  narysuj dowolny trójkąt. Następnie korzystając z opcji **Construction** wyznacz środki S1 i S2 dowolnych dwóch boków tego trójkąta. Z kolei połącz odcinkiem punkty S1 i S2 . Teraz zmieniaj długości boków (kąty) tego trójkąta.

1. Zapisz prawidłowość, którą zauważasz odnośnie położenia odcinka S1S2 w stosunku do boku przeciwległego tego trójkąta?
2. Zmierz długość odcinka S1S2 oraz długość boku przeciwległego tego trójkąta. Korzystając z opcji **Action** i **Calculate** wyznacz stosunek długości tych odcinków. Następnie zmieniaj długości boków (kąty) trójkąta. Jaką prawidłowość zauważasz? Zapisz tę prawidłowość.

**Rozwiązanie: a)**

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**Odpowiedź: a)** ………………………………………………………………………………….

**Rozwiązanie: b)**

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**Odpowiedź: b)** ………………………………………………………………………………..

**ZADANIE 6 (3pkt)**

Oblicz sumę trzydziestej drugiej, trzydziestej piątej i czterdziestej drugiej cyfry występujących po przecinku w rozwinięciu dziesiętnym liczby $\frac{1}{39}$.

**Rozwiązanie:**

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**Odpowiedź:** …………………………………………………………………………………….

**ZADANIE 7 (5pkt)**

Długość jednego boku prostokąta skrócono o 1%, a długość drugiego boku tego prostokąta zwiększono o 1%

**a)** Oznacz długości boków wyjściowego prostokąta przez *a* i *b* wyznacz pole prostokąta po zmianie długości tych boków. Wynik podaj w najprostszej postaci.

**b)**  Obliczoile procent zmieni się pole prostokąta po zmianie boków. Pole to będzie większe czy mniejsze w stosunku do pola prostokąta wyjściowego?.

**Rozwiązanie a):**

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**Odpowiedź a):** …………………………………………………………………………………………………...

**Rozwiązanie b):**

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**Odpowiedź b):**

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**ZADANIE 8 (7pkt)**

Korzystając z okna **Graphs** i opcji **Geometry** narysuj okrąg o środku w punkcie A = (0,0) i promieniu długości 5. Następnie narysuj okrąg o środku w punkcie B = (2,2) przechodzący przez punkt C = (8,3). Okręgi te przecinają się w punktach P i Q.

1. Wyznacz pole trójkąta BPQ. Wynik podaj z dokł. do jednego miejsca po przecinku.
2. Wyznacz obwód trójkąta BPQ. Wynik podaj z dokł. do dwóch miejsc po przecinku.
3. Podaj współrzędne punktu S, który jest środkiem odcinka BC.

**Rozwiązanie a):**

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**Odpowiedź a):** ……………………………………………………………………………..

**Rozwiązanie b):**

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**Odpowiedź b):** ……………………………………………………………………………..

**Rozwiązanie c):**

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**Odpowiedź c):** ………………………………………………………………………………….

**ZADANIE 9 (6pkt)**

Znajdź dwie kolejne liczby naturalne *a* i *b*, których suma drugich potęg jest równa 29525

**Rozwiązanie:**

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**Odpowiedź:** *a* = ……………………………., *b* = …………………………………………

**ZADANIE 10 (4pkt)**

Dany jest wyrażenie*:*

$$16∙\left(\frac{1}{5}-\frac{1}{3}∙\frac{1}{5^{3}}+\frac{1}{5}∙\frac{1}{5^{5}}-\frac{1}{7}∙\frac{1}{5^{7}}+…+(-1)^{n}\frac{1}{2n+1}∙\frac{1}{5^{2n+1}}\right)-4∙(\frac{1}{239}-\frac{1}{3}∙\frac{1}{239^{3}}+\frac{1}{5}∙\frac{1}{239^{5}}+…+(-1)^{n}∙\frac{1}{2n+1}∙\frac{1}{239^{2n+1}})$$

dla wszystkich liczb naturalnych *n*.

1. Wyznacz wartość tego wyrażenia dla *n* = 0, *n* = 1, *n* = 2 i *n* = 3 z dokł. do dwóch miejsc po przecinku.
2. Obserwując zmianę wartości wyrażenia z podpunktu **a)** sformułuj hipotezę dotyczącą wartości wyrażenia w przypadku, gdy liczba *n* będzie nieskończenie duża ($\infty $). Jest to bardzo znana stała.

Rozwiązanie: **a)**

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**Odpowiedź: a)** *a*(0) = ………., *a*(1) = …………, *a*(2) = ………….., *a*(3) = ………………...

**Odpowiedź: b)** ……………………………………………………………………………….